Shear Viscosity of the "semi"-QGP

1. Deconfinement and Polyakov loops: possible phase transitions

2. SU(∞) on a small sphere (Sundborg '99, Aharony et al '03, '05):
   Matrix model; Gross-Witten point; semi-QGP

3. Lattice: pressure for SU(N)
   With quarks: flavor independence. Without quarks: N = 3 like N = ∞
   Is the QCD coupling big at T_c? Maybe *not*.

4. Renormalized Polyakov Loops & the semi-QGP

5. Shear viscosity of the semi-QGP

For heavy ions, is LHC like RHIC?

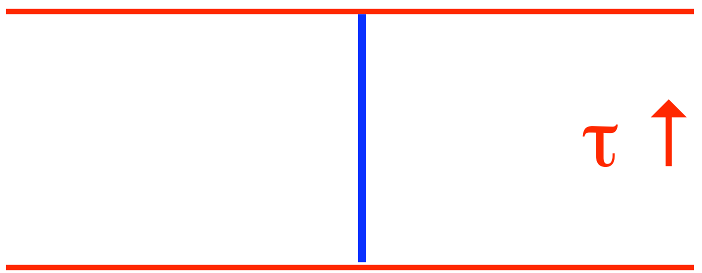
Strong-QGP, $\mathcal{N} = 4$ SUSY: *yes*.
Semi-QGP, *no*.

1. Some possible deconfining transitions

Polyakov loops & deconfinement

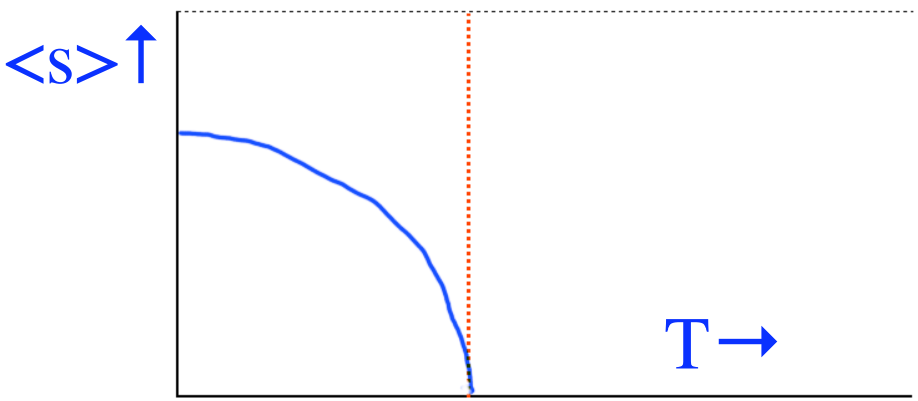
Polyakov loop: order parameter for deconfinement in SU(N):

$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathcal{P}\;\exp\left(ig\int_0^{1/T} A_0\,d\tau\right)$$



Ordinary magnetization:

<s>≠ 0 at low T, <s>=0 at high T.



Deconfinement: Polyakov loop "flipped",

Global Z(N) symmetry: *broken* at high T,

restored at low T.

Classify possible deconfining transitions by change in < *loop* >.

Assume overall normalization of loop physical: $\langle \ell \rangle \to 1 \;\;,\; T \to \infty$

Quarks act like background Z(N) field.

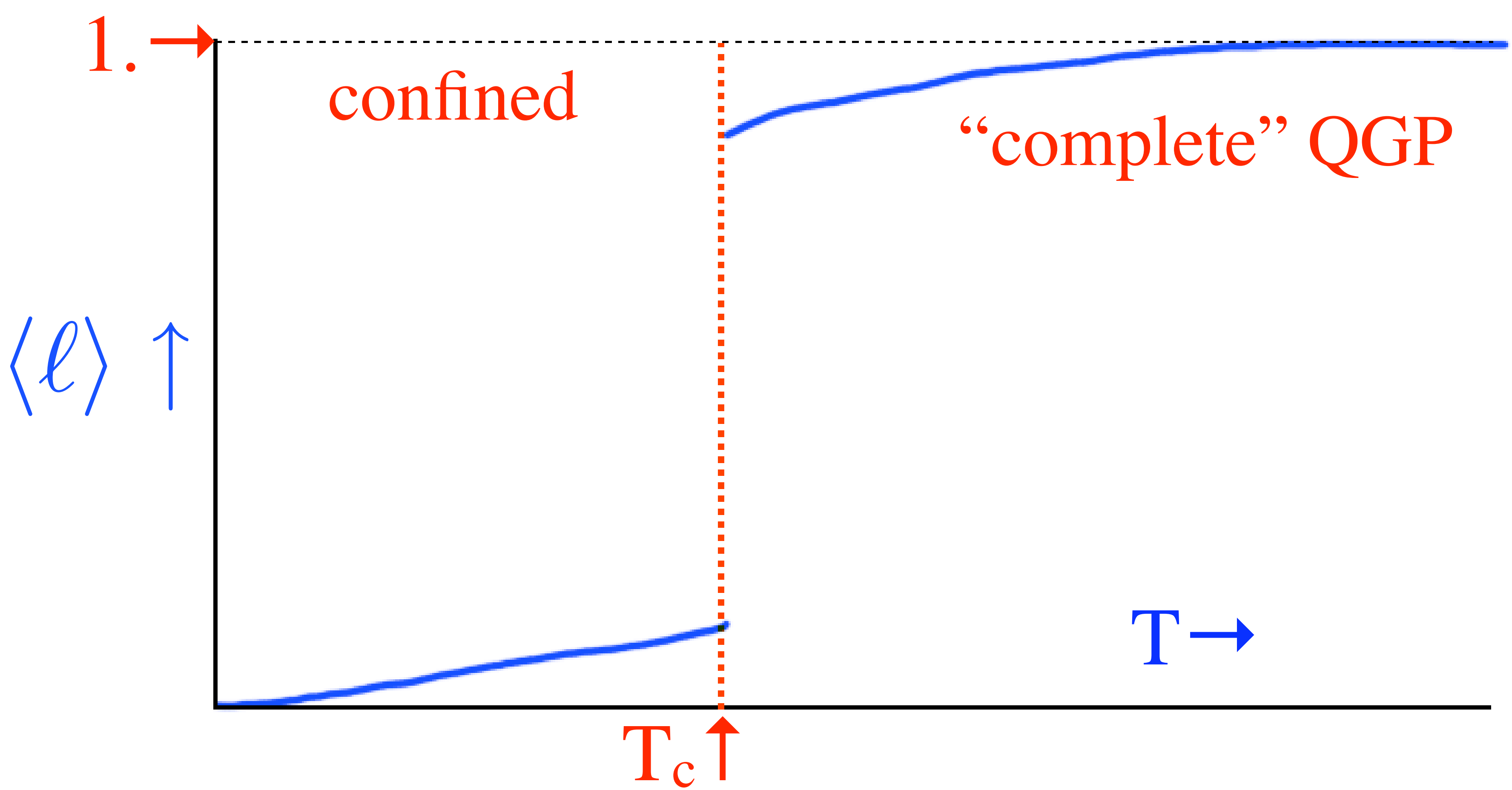
Consider order parameter, *not* pressure, p(T); pressure always continuous.

One possibility

Transition from confined phase to "complete" Quark-Gluon Plasma (QGP)

Complete QGP: loop near 1, ≈ perturbative.

Transition strongly first order. Effect of quarks weak.



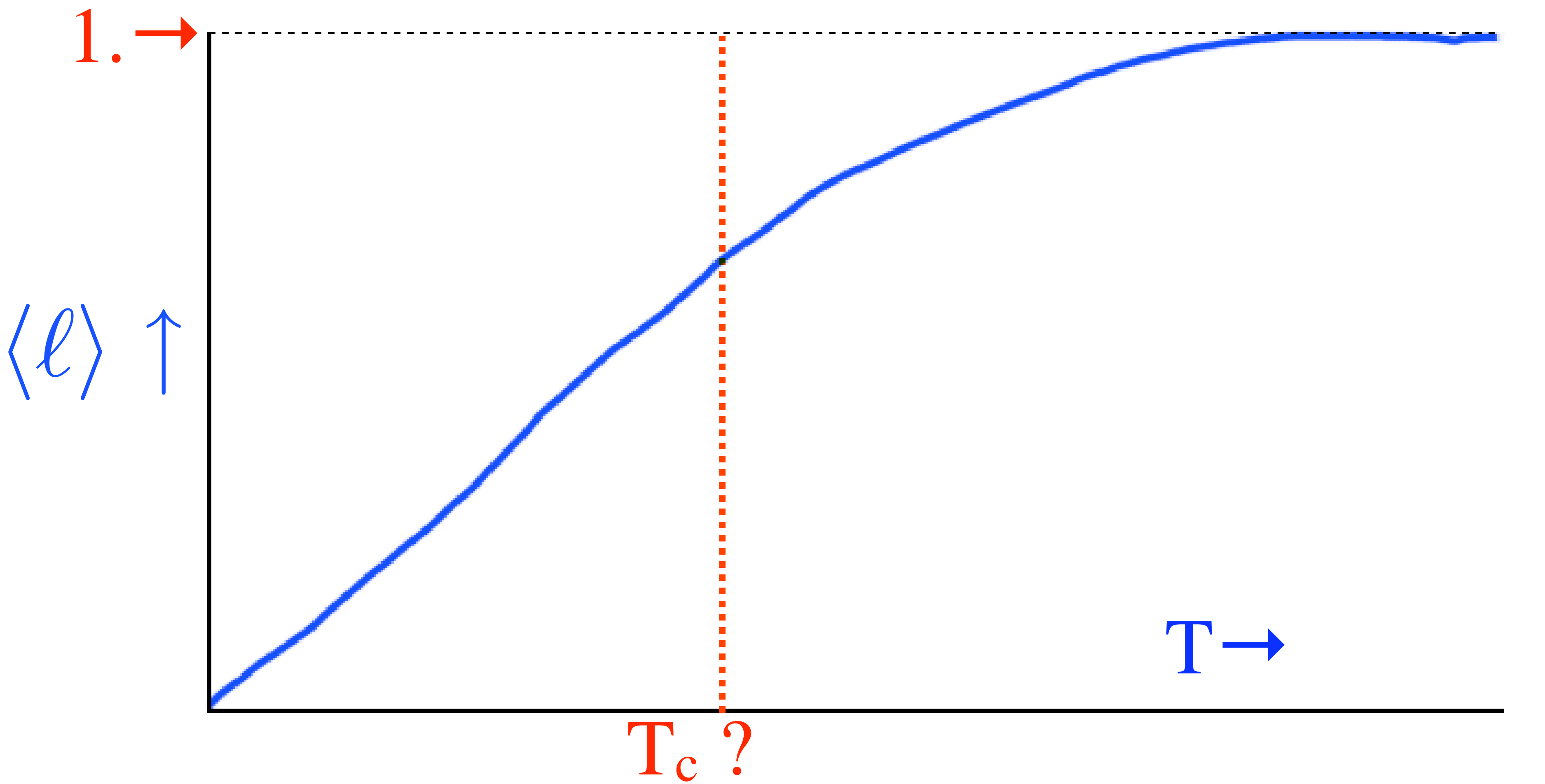
Logically possible, does *not* appear to arise in *any* context. (Lattice, analytical...)

General expectation before RHIC.

Another possibility

Many quarks, *strong* background field.

Loop increases gradually, probably no deconfining phase transition.



Probably true for large number of flavors, completely wash out deconfinement.

Also: perhaps no chiral transition?

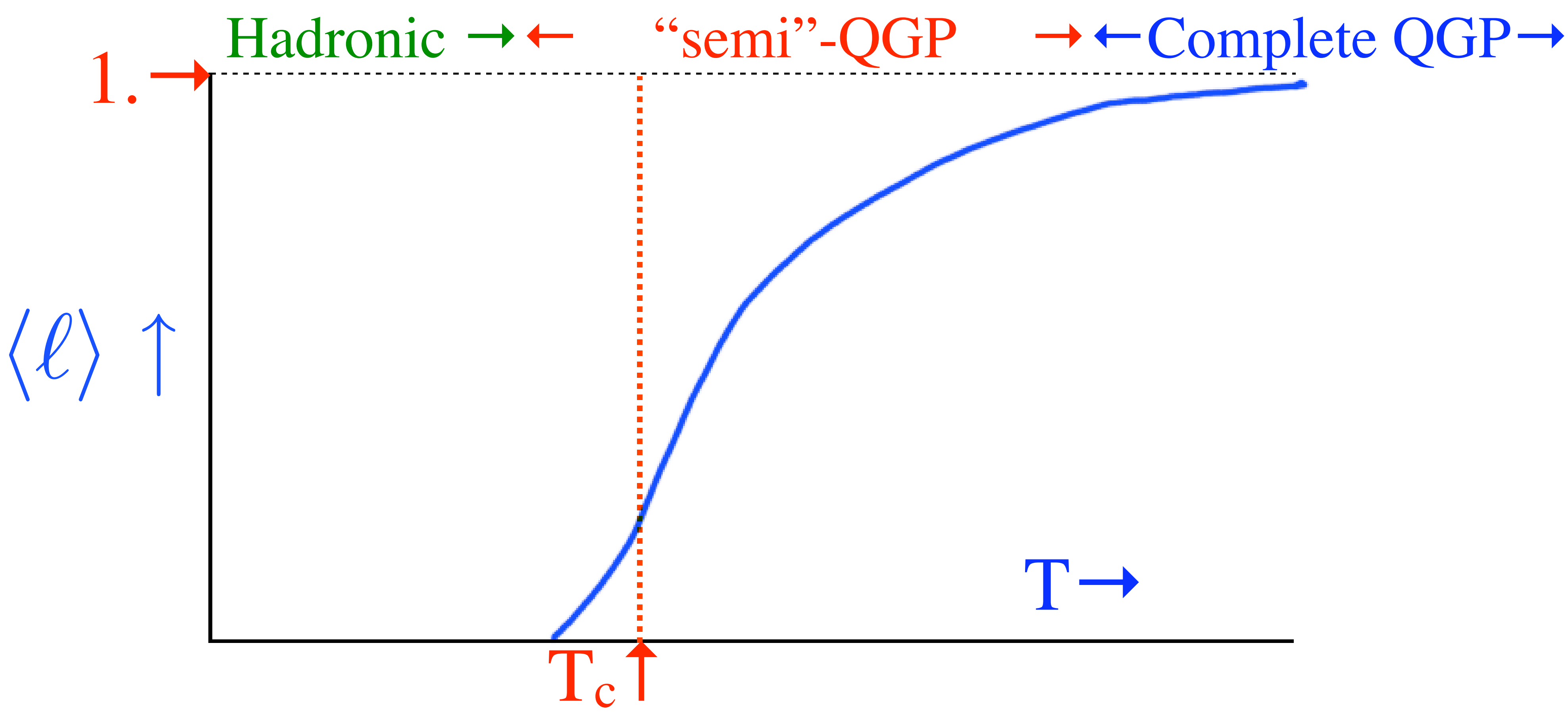
QCD?

Analytic solution, and lattice, show: even with dynamical quarks, *three* regimes:

Hadronic, <*loop*> ~ 0.

"Semi"-QGP: <*loop*> nonzero, but *not* near one. Matrix model.

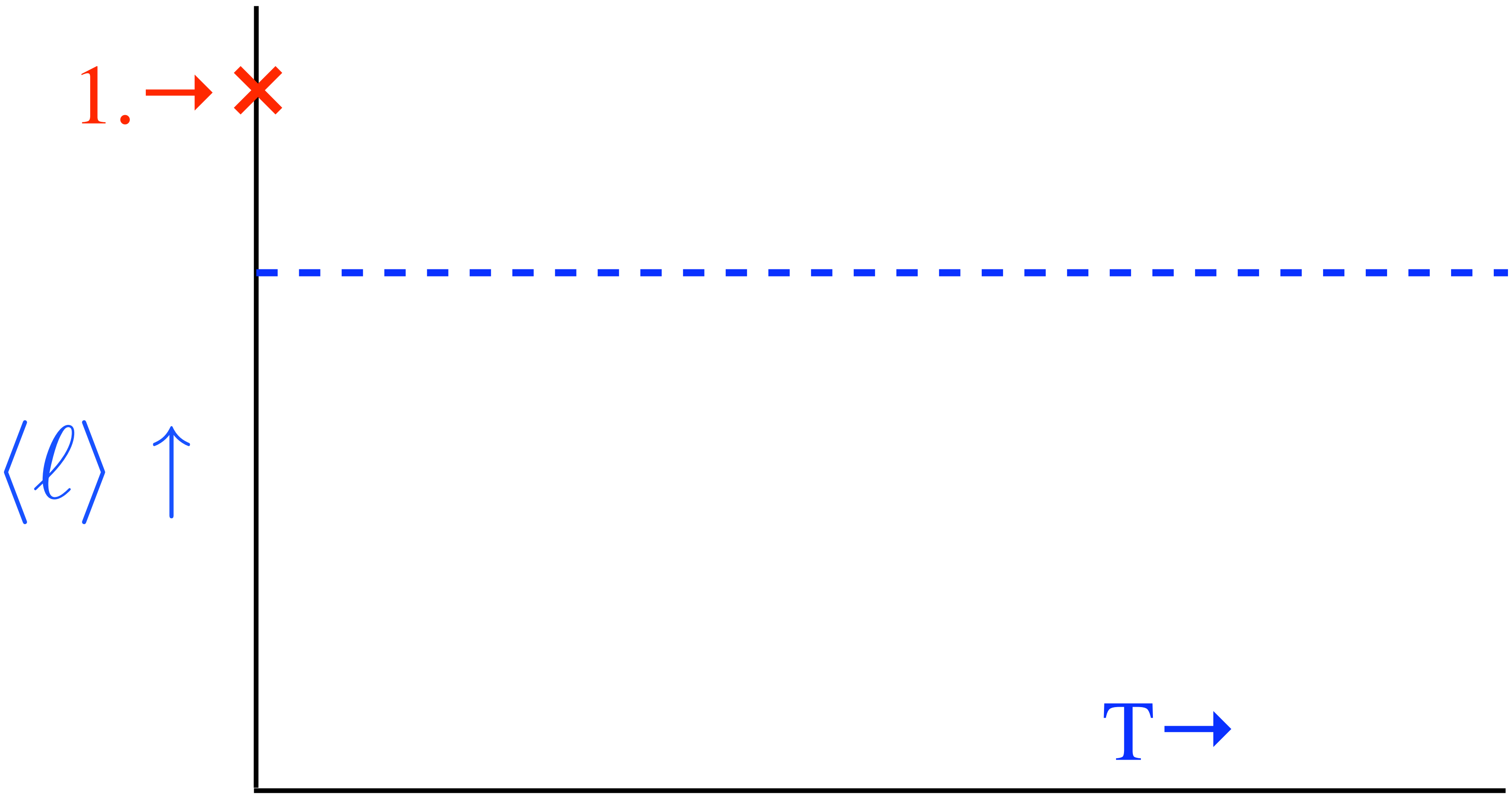
Complete QGP: <*loop*> near one. Usual "perturbative" regime (resummed!)



$\mathcal{N} = 4$ SU(∞)

AdS/CFT: Can define <loop> = 1 at T = 0 (Polyakov-Maldacena, + scalars)

Constant at $T \neq 0$ (like pressure/T^4) : value, vs g^2 N? *Not* a deconfining transition.



2. Deconfinement for SU(∞) on a small sphere

SU(∞) on a small sphere: Hagedorn temperature

Sundborg, hep-th/9908001
AMMPV: Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk,
hep-th/0310285 & 0502149

Consider SU(N) on a *very* small sphere: radius R, with $g^2(R) << 1$.
(Sphere because constant modes simple, spherically symmetric)

At $N = \infty$, can have a phase transition even in a *finite* volume.

When $g^2 = 0$: by counting gauge *singlets*, find a Hagedorn temperature, T_H :

$$\rho(E) \sim \exp(E/T_H) \;\; , \;\; E \to \infty$$

At $N = \infty$, Hagedorn temperature is *precisely* defined, calculable at $g^2 = 0$

$$T_H = \frac{1}{\log(2+\sqrt{3})} \, \frac{1}{R} \;\; , \;\; g^2 = 0.$$

SU(∞) on a small sphere: effective theory

Construct effective theory for low energy (constant) modes,
by integrating out high energy modes, with momenta ~ 1/R:

Consider (thermal) Wilson line:

$$\mathbf{L} = \mathcal{P}\ \exp\left(ig \int_0^{1/T} A_0 \, d\tau \right)$$

L is gauge dependent,

$$\mathbf{L} \rightarrow \Omega(1/T)^\dagger \, \mathbf{L} \, \Omega(0)$$

Traces of moments gauge invariant,

$$\ell_j = \frac{1}{N} \operatorname{tr} \mathbf{L}^j \quad , \quad j = 1 \ldots (N-1)$$

Effective theory for l_j: compute free energy in *constant* background A_0 field:
Q = diagonal matrix.

$$A_0 = \frac{T}{g}\, Q \quad , \quad \mathbf{L} = \mathrm{e}^{iQ}$$

SU(∞) on a small sphere & the Polyakov loop

When $g^2 = 0$:

$$\mathcal{V}_{eff} = N^2 \left(m^2 \, \ell_1^2 + \mathcal{V}_{\rm Vdm} + \ldots \right) \quad ; \quad m^2 \sim T_H^2 - T^2$$

At the Hagedorn temperature, T_H, *only* the first mode, l_1, is unstable; all other modes are stable. Concentrate on that mode, $l \equiv l_1$.

Vandermonde determinant in measure for constant mode gives "Vdm potential":

$$\mathcal{V}_{\rm Vdm} = + \, \ell^2 \ , \ \ \ell < \frac{1}{2}$$

$$\mathcal{V}_{\rm Vdm} = - \, \frac{1}{2} \, \log \left(2 \, (1 - \ell) \right) + \frac{1}{4} \ , \ \ \ell \geq \frac{1}{2}$$

Vdm potential has discontinuity of *third* order at $l = 1/2$.

Gross & Witten '81; Kogut, Snow & Stone '82....

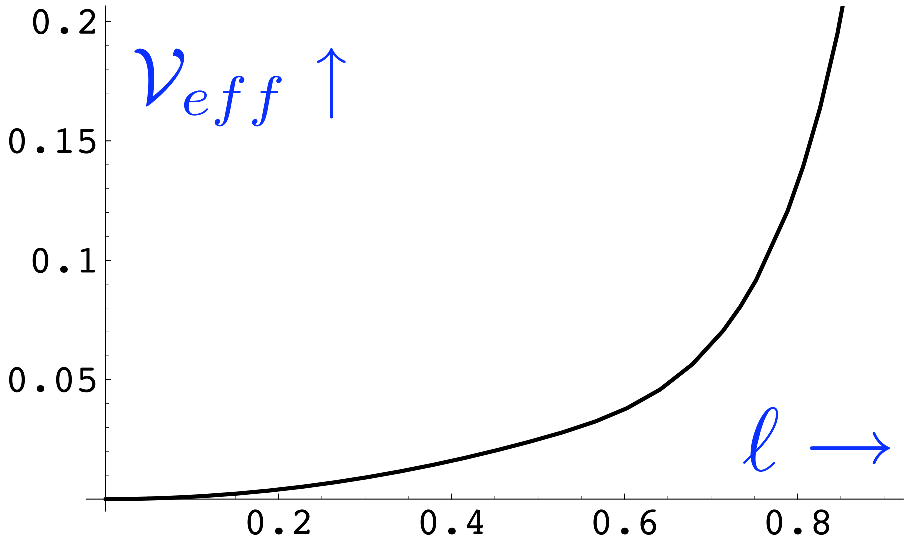
Sundborg, '99....AMMPV '03 & '05

Dumitru, Hatta, Lenaghan, Orginos & RDP, hep-th/0311223 = DHLOP

Dumitru, Lenaghan & RDP, hep-ph/0410294.

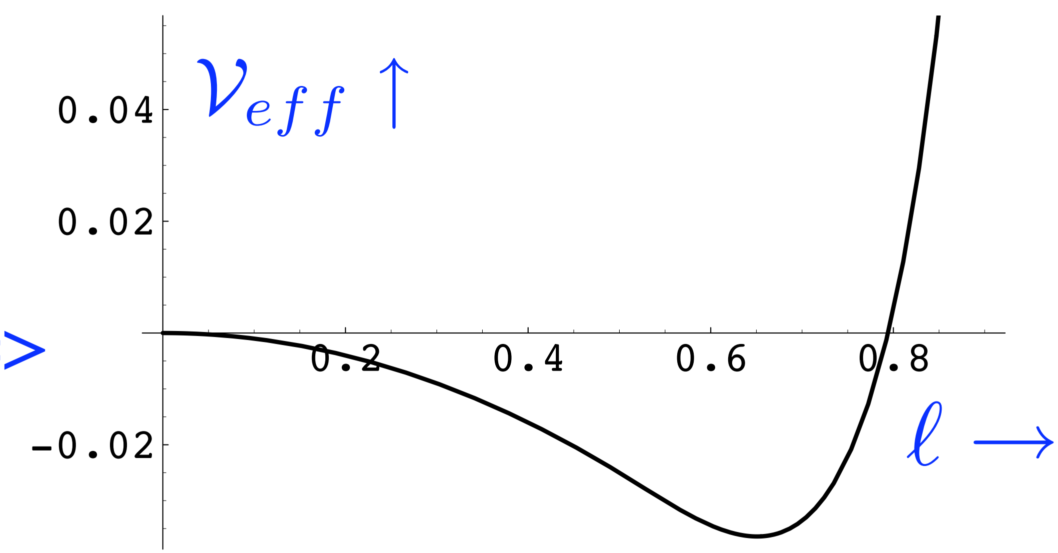
Deconfinement on a small sphere

Have deconfining phase transition when $m^2 = 0$: *first* order, $< l > = 1/2$ at $T_c = T_H$.
Obvious from potentials above and below T_c:



<= $m^2 = +.1$, confined phase

$m^2 = -.1$, deconfined phase =>



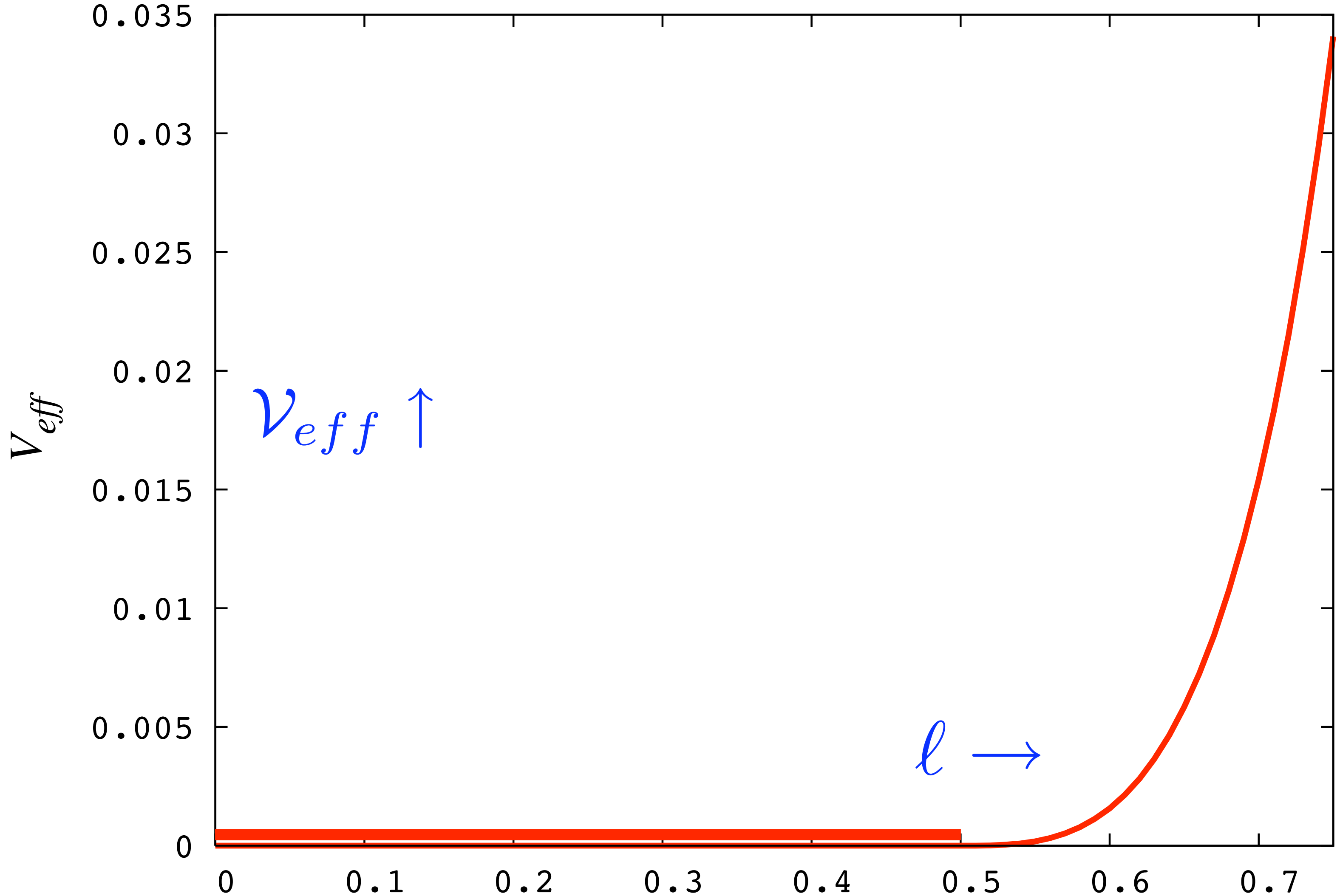
Gross-Witten point

At transition, order parameter <*loop*> jumps from 0 to 1/2. Latent heat nonzero.

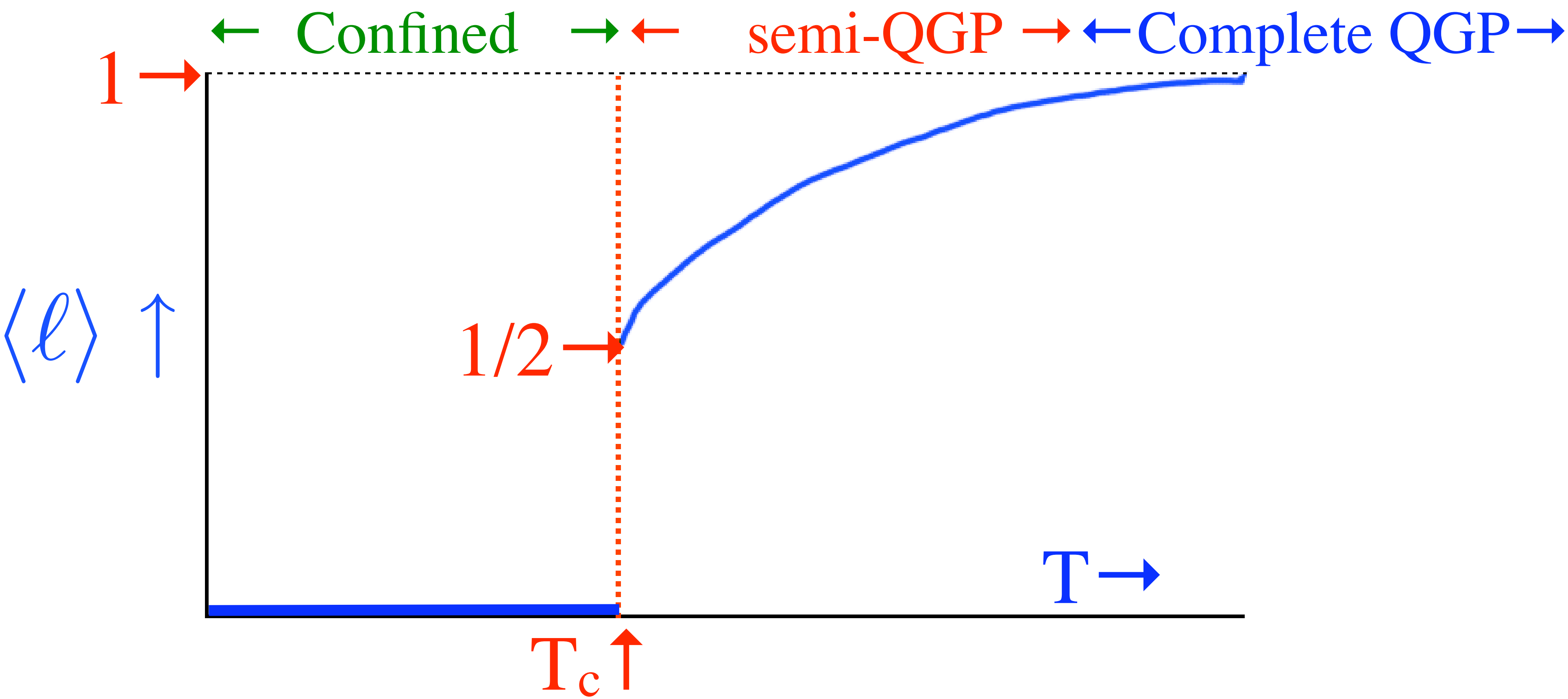
DLP: masses vanish, asymmetrically: "critical" 1st order transition: "GW point".

At $m^2 = 0$, <*loop*> jumps because of 3rd order discontinuity in Vdm potential

GW point like tricritical point in extended phase diagram.



Semi-QGP on a small sphere



Boundary btwn complete and semi-QGP *not* precise; <*loop*> → 1 by T ~ # T_c?

To higher order in g^2 :

$$\mathcal{V}_{eff} = \mathcal{V}_{eff}(g^2 = 0) - c_3\, g^4 \left(\ell^2\right)^2 \quad c_3 > 0.$$

AMMPV '05: calculate free energy with Q ≠ 0 to *two* loop order at small R

$c_3 > 0 \Rightarrow T_c = T_H - O(g^4)$. Deconfinement first order, *below* T_H

3. Lattice: pressure. $N = 3$ like $N = \infty$?

Maybe α_s is *not* so big at T_c

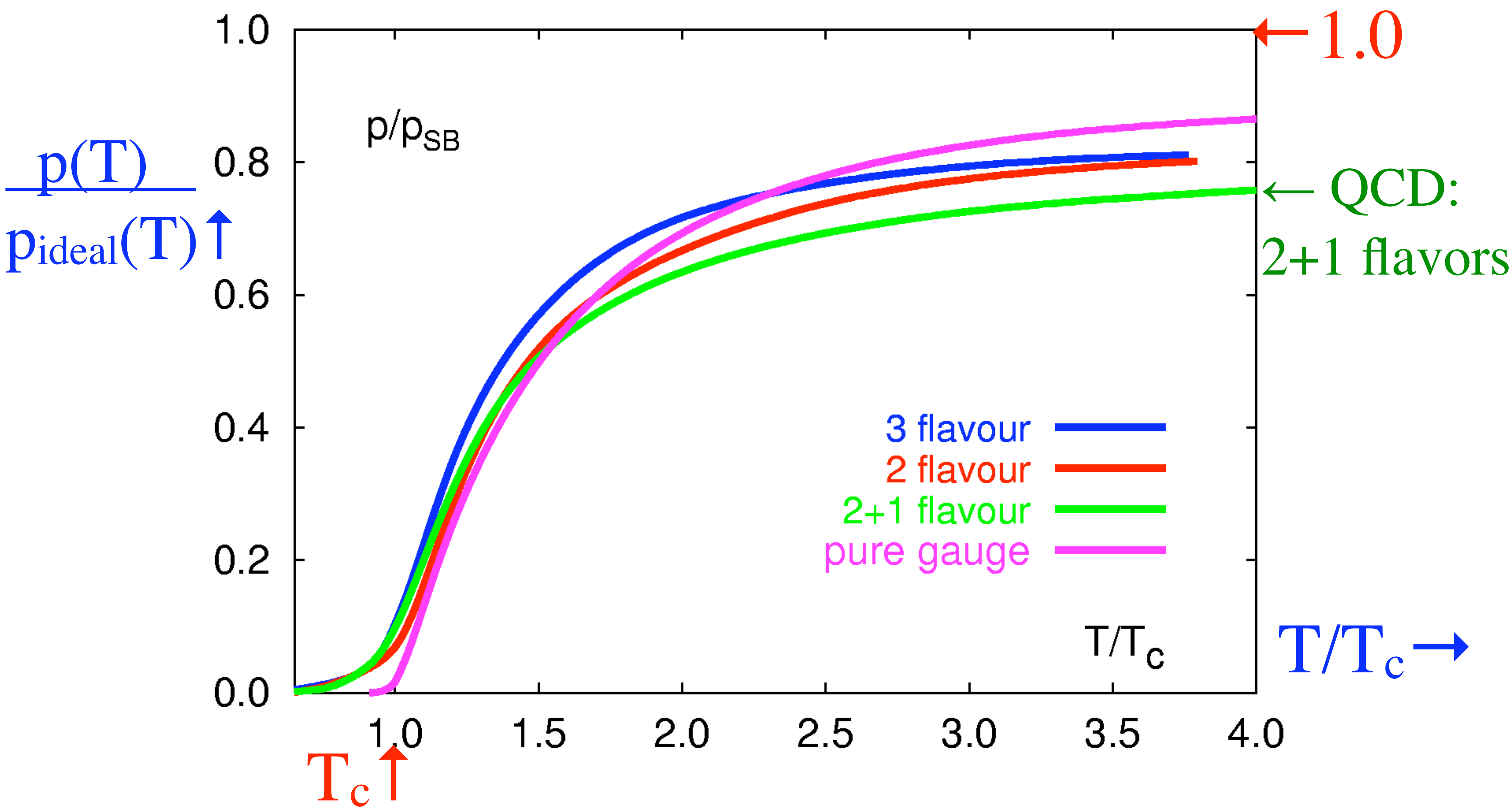
Lattice: pressure & “flavor independence”

Pure SU(3): *weakly* 1st order

QCD: and 2+1 flavors: crossover

Bielefeld: properly scaled, $\approx$ *universal* pressure

$$\frac{p}{p_{ideal}}\left(\frac{T}{T_c}\right) \approx \text{const.}$$

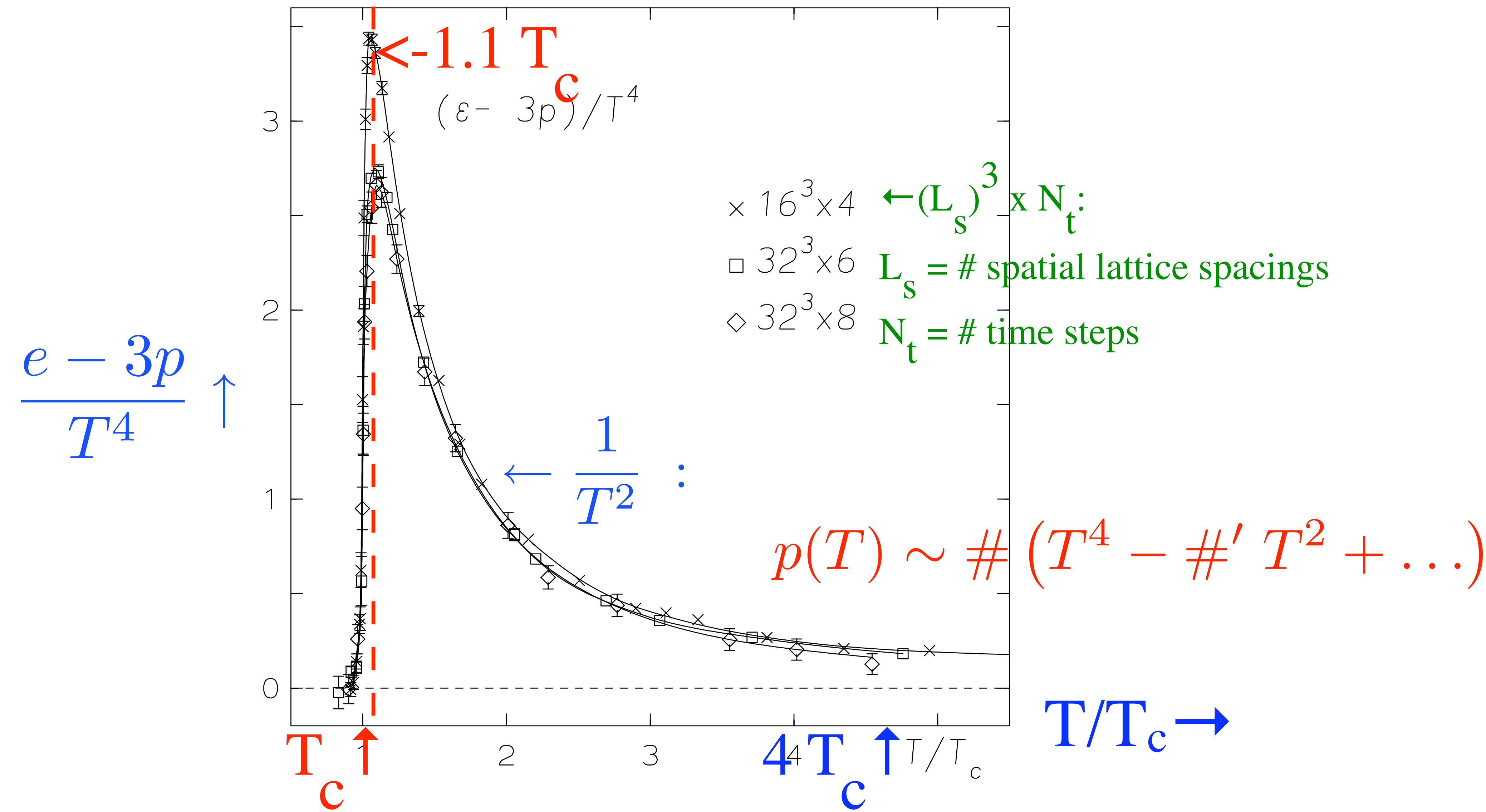


Lattice: SU(3) glue, no quarks

More sensitive than pressure: $(e\text{-}3p)/T^4$, e = energy density, p = pressure

Bielefeld, hep-lat/9602007. N_t = # time steps: 6, 8 near continuum limit?

Pressure: sum of ideal gas, T^4, plus T^2 , *then* "MIT bag constant", T^0.



Lattice: SU(3) close to SU(∞)?

Bringoltz & Teper, hep-lat/0506034 & 0508021:

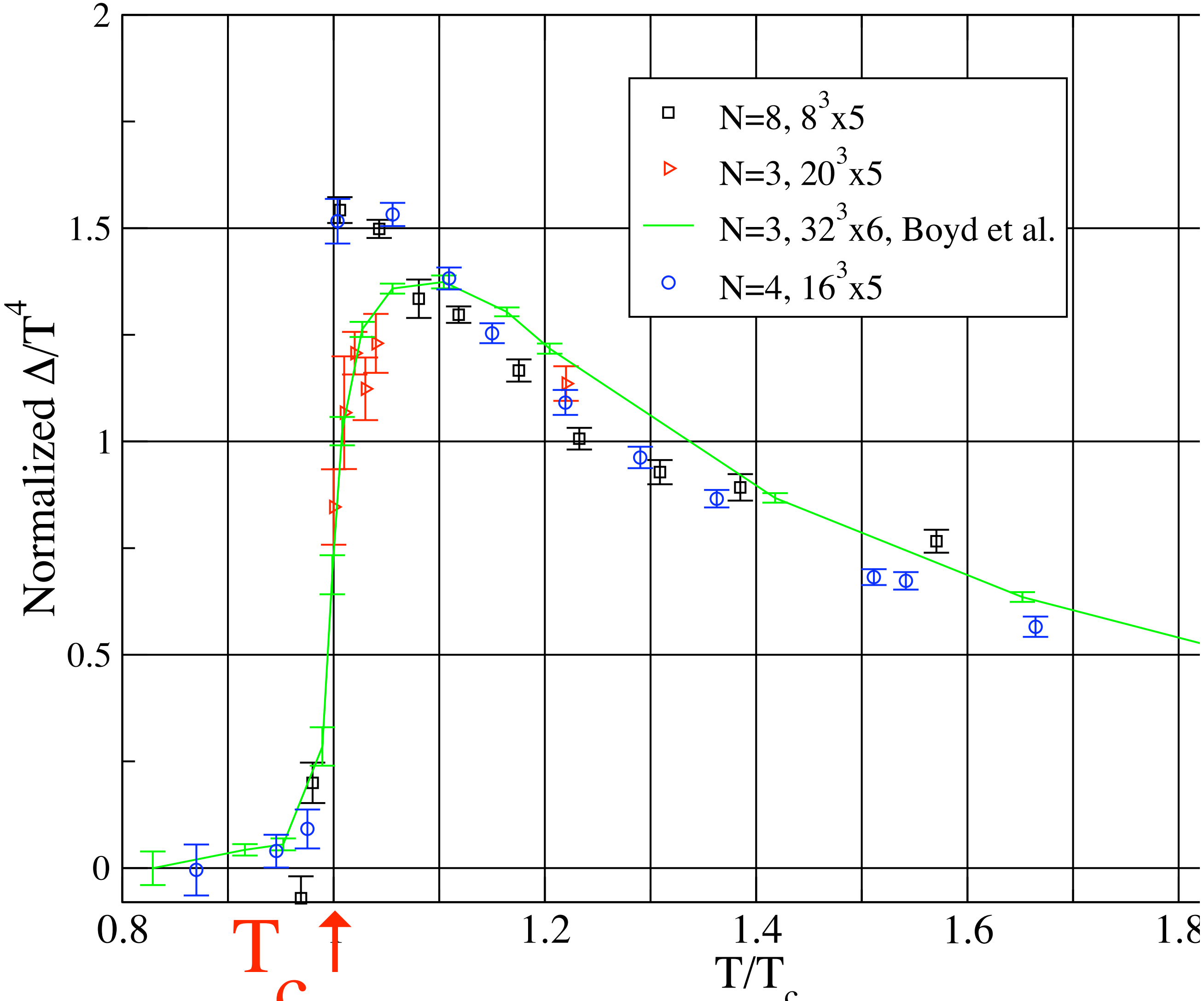
SU(N), no quarks, N= 3, 4, 6, 8, 10, 12.

Deconfining transition first order, latent heat ~ N^2.

Hagedorn temperature $T_H \sim 1.116(9)\, T_c$ for $N = \infty$

$$\frac{e-3p}{N^2\, T^4} \sim \text{const.}$$

$$\frac{e-3p}{N^2\, T^4} \uparrow$$



$T_c \uparrow$

$T/T_c \rightarrow$

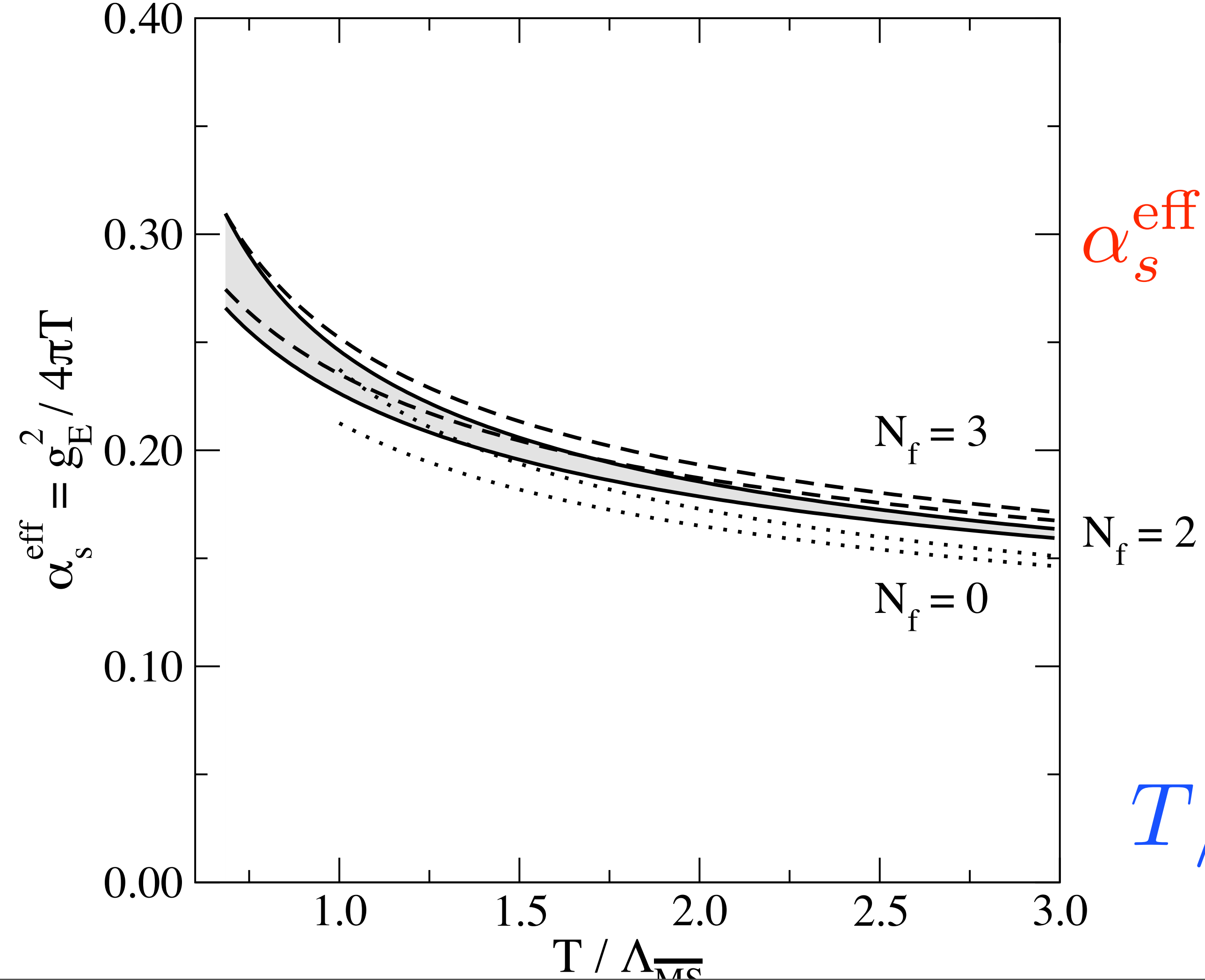
Maybe α_s is *not* so big at T_c

Laine & Schröder, hep-ph/0503061 & 0603048

$T_c \sim \Lambda_{MS} \sim 200$ MeV. But $\alpha_s^{eff}(T) \sim \alpha_s^{eff}(2\pi T) \sim 0.3$ at T_c: *not* so big

Two loop calculation: grey band uncertainty from changing scale by factor 2.

$\alpha_s^{\rm eff}(T) \uparrow$



$\alpha_s^{\rm eff}(T_c) \approx 0.3$

$T/\Lambda_{\overline{\rm MS}} \rightarrow$

Perturbative resummation of the pressure

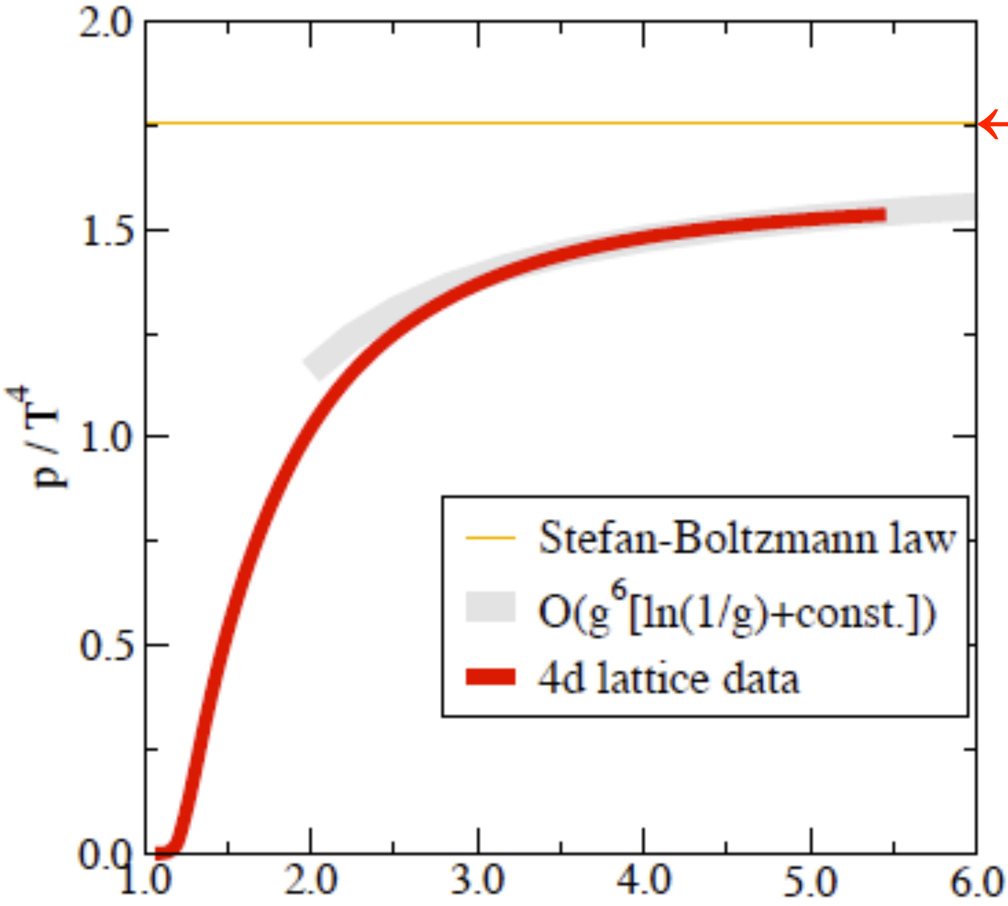
"Helsinki" resummation: Di Renzo, Laine, Schröder, Torrero, 0808.0557

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 + \; m_D^2\,\mathrm{tr}\,A_0^2 \;+\; \kappa\,\mathrm{tr}\,A_0^4$$

Now to 4 loop, ~ g^6. *Works* to ~ 3 T_c, *fails* below.

Why, if $\alpha_s^{eff}(T_c)$ is *not* so big? Perhaps a *semi*-QGP near T_c?

$p/T^4 \uparrow$



←ideal gas

Grey: resummed pert.theory.

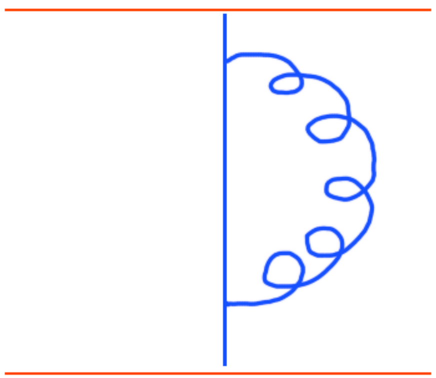
Red: lattice.

$T/\Lambda_{\overline{MS}} \rightarrow$

4. Renormalized Polyakov loops & semi-QGP

Renormalized loops

Polyakov '80, Dotsenko & Vergeles '81...DHLOP '03...
Gupta, Hubner & Kaczmarek 0711.2251 = GHK



Bare loop UV divergent. At one loop =>
Like mass ren. of heavy quark. In 3+1 dim.'s, linear div.
Vanishes with dimensional regularization, but *not* on the lattice:

$$\langle \ell_R \rangle - 1 \sim \# \frac{C_R g^2}{T} \int^{1/a} \frac{d^3k}{k^2} = \# \left(C_R\, g^2 + \#' g^4 + \ldots \right) \frac{1}{aT}$$

Loop in representation R, Casimir C_R.
1/(a T) = # time steps, N_t. Renormalized loop:

$$\ell_R^{\rm bare} = \mathcal{Z}_R(g^2)^{N_t}\; \ell_R^{\rm ren}$$

Can choose $\langle \ell \rangle \to 1 \quad , \; T \to \infty$

GHK: find approximate Casimir scaling:
Like cusp anomalous dimension.

$$\mathcal{Z}_R(g^2) \approx \mathcal{Z}(g^2)^{C_R}$$

Zero point energy & renormalized loops

Renormalization valid for arbitrary Wilson loops:

$$\mathcal{W} = \mathrm{tr}\, \mathcal{P}\, e^{ig \oint A_\mu dx^\mu} \quad ; \quad \mathcal{W}_{\mathrm{bare}} = \mathcal{Z}_{\mathrm{div}}\, \mathcal{W}_{\mathrm{ren}}$$

Two ambiguities:

$$\mathcal{Z}_{\mathrm{div}} = \mathrm{e}^{E_0 L}\, \mathcal{Z}_0\, \mathcal{Z}(g^2 \ldots)^{L/a} \quad ; \quad \mathcal{W}_{\mathrm{ren}} \rightarrow \mathrm{e}^{-E_0 L}\, \mathcal{Z}_0^{-1}\, \mathcal{W}_{\mathrm{ren}}$$

Overall scale trivial: $Z_0 = 1$ by requiring < *loop* > → 1 as T →∞.

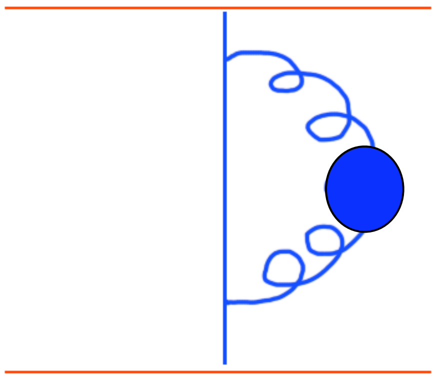
E_0 = ground state energy for potential from Wilson loop: $E_0 = \# \sqrt{\sigma}$. # ?

E_0 : can *define* = 0 order by order in perturbation theory with *any* regulator
(Obvious with dimensional reg.. Also true with higher derivatives...)
$E_0 = 0$ also in string model: Nambu-Goto *plus* extrinsic curvature terms...

Lattice provides *non*-perturbative way to *define* $E_0 = 0$. Still, its a choice...

T = 0 potential with dynamical quarks: can define *energy* for string breaking?

Renormalized loops at high T



Gava & Jengo '81:

Renormalized loops approach unity from *above*.

Can compute perturbatively, with dimensional regularization.

Just fold Debye mass, m_D , into propagator for A_0:

$$\langle \ell_R^{\rm ren} \rangle - 1 \sim (-)\, \frac{C_R\, g^2}{T} \int d^3k\, \frac{1}{k^2 + m_{\rm D}^2} \sim (-)\, \frac{C_R\, g^2}{T}\, (-)\, \sqrt{m_{\rm D}^2}$$

Sign of the integral is *negative*; like subtracting $1/k^2$ propagator.

$$\langle \ell_R^{\rm ren} \rangle - 1 \sim (+)\, \frac{C_R}{N}\, \frac{(g^2\, N)^{3/2}}{8\pi\sqrt{3}}$$

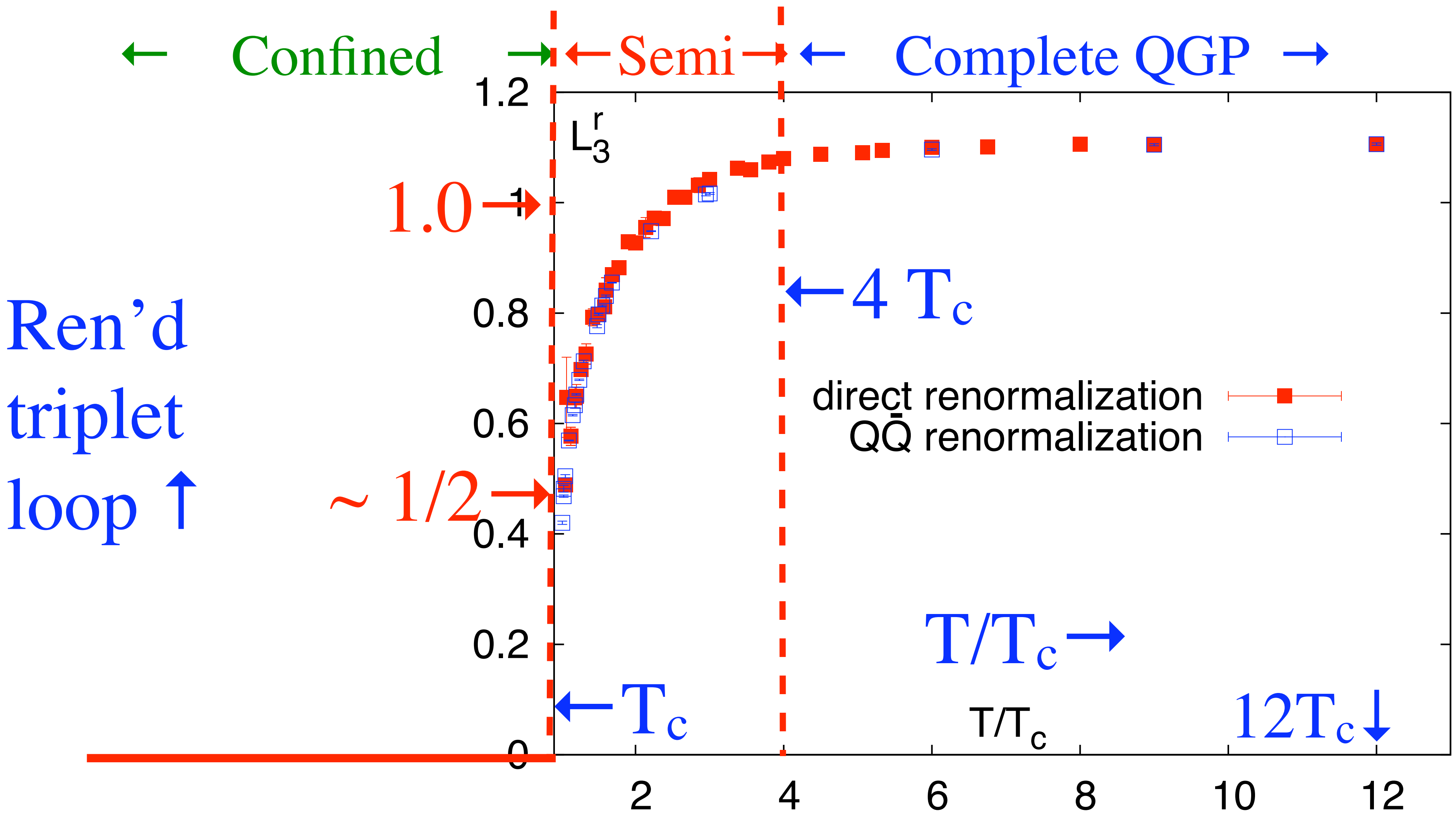
Lattice: ren.'d triplet loop, pure SU(3)

GHK: Lattice SU(3), *no* quarks. Two ways of getting ren'd loop agree.

<*triplet loop*> ~ 1/2 at T_c^+! N=3 close to Gross-Witten point?

<*adjoint loop*> ~ 0.01 just below T_c . *Only* natural in matrix model.

semi-QGP: from (*exactly*) T_c^+ to 2 - 4 T_c (?). <*loop*> ~ constant above 4 T_c.



Lattice: renormalized loop *with* quarks

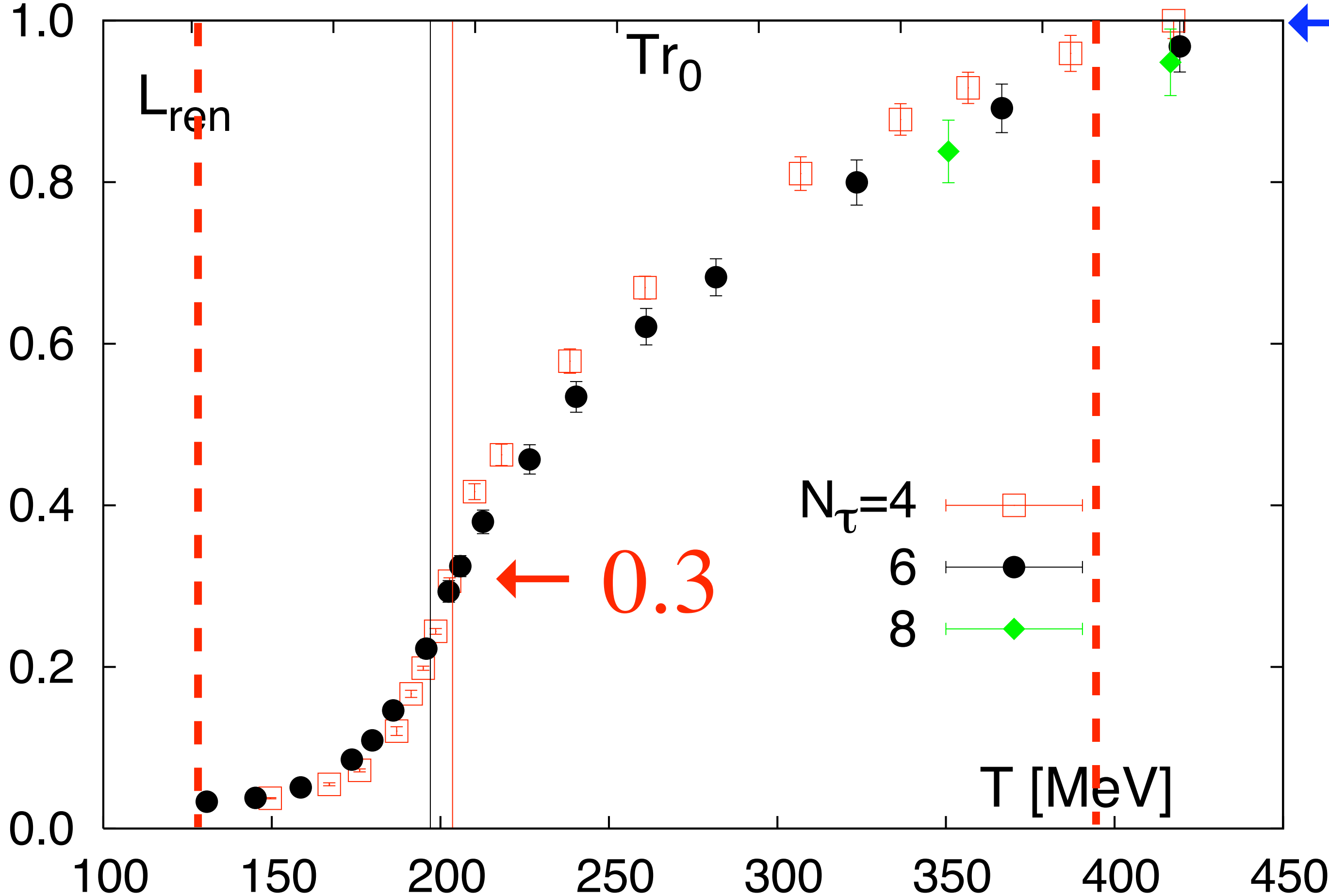
Cheng et al, 0710.0354: ~ QCD, 2+1 flavors. T_c ~ 190 MeV, crossover.

<*loop*>: nonzero from ~ 0.8 T_c; ~ 0.3 at T_c; ~ 1.0 at 2 T_c.

Semi-QGP from ~ 0.8 T_c (*below* T_c) to ~ 2-3 T_c (?). <*loop*> *small* at T_c .

← Confined →← Semi-QGP →←Complete QGP

Ren.'d triplet loop ↑



← 1.0

T→

.8 T_c↑ T_c↑ 2 T_c↑

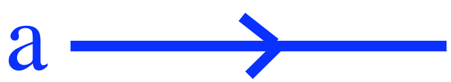
4. Shear viscosity of the semi-QGP

Semi-QGP in weak coupling

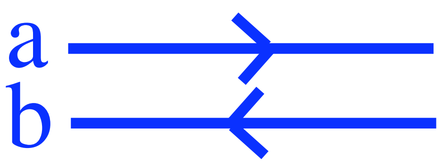
Hidaka & RDP 0803.0453. Semi-classical expansion of the semi-QGP:

$$A_\mu = A_\mu^{\rm cl} + B_\mu \quad , \quad A_0^{\rm cl} = Q/g \quad .$$

$Q \neq 0$: just like semi-classical calc. of ‘t Hooft loop. $Q = Q^a$, *diagonal* matrix.
Work at large N, large N_f, use double line notation. (Finite N ok, messy.)



$$i D_0^{\rm cl} = p_0 + Q^a = p_0^a$$



$$i D_0^{\rm cl} = p_0 + Q^a - Q^b = p_0^{ab}$$

Perturbation theory in B_μ’s same as $Q = 0$, but with “shifted” p_0’s.
Amplitudes in real time: $p_0^a \rightarrow i\,\omega$, etc. Furuuchi, hep-th/0510056

Q (imaginary) chemical potential
for (diagonal) color charge.
e.g., for quarks:

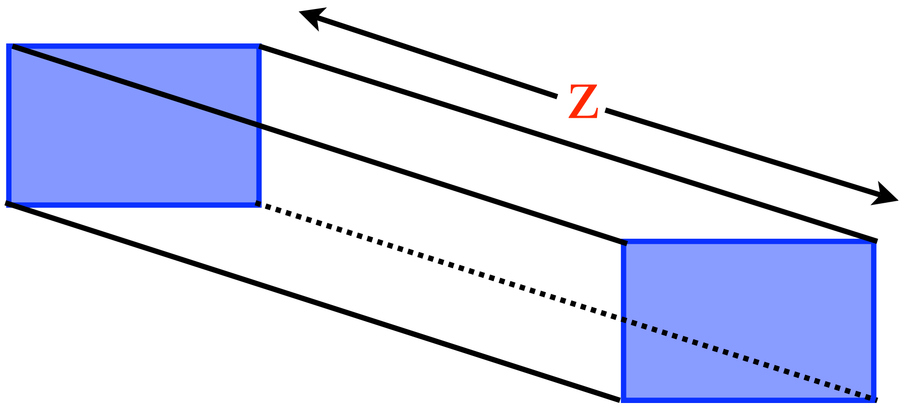
$$\widetilde{n}(E - iQ^a) = \frac{1}{{\rm e}^{(E-iQ^a)/T} + 1}$$

Z(N) interfaces = ‘t Hooft loop

Z(N) interface: Z(N) “twist” in z-direction. A_{tr} = transverse area.

$$A_0^{\rm cl} = \frac{2\pi T}{gN}\; q(z)\; t_N$$

$\langle L \rangle = \mathbf{1}$



$\langle L \rangle = \mathrm{e}^{2\pi i/N}\,\mathbf{1}$

$t_N = \mathrm{diag}(1_{N-1}, -N+1)$. $A_0 \sim$ “coordinate” q(z).

$L_{\rm eff}$ = classical + 1 loop potential, for *constant* A_0

$$\mathcal{L}_{\rm eff} = \frac{4\pi^2 (N-1) T^3}{\sqrt{3g^2 N}}\, \mathrm{A_{tr}} \int dz \left(\left(\frac{dq}{dz} \right)^2 + q^2 (1-q)^2 \right)$$

Bhattacharya, Gocksch, Korthals-Altes & RDP, hep-ph/9205231

Z(N) interface = ‘t Hooft loop: Korthals-Altes, Kovner & Stephanov, hep-ph/9909516

Corrections ~ g^3: Giovannangeli & Korthals-Altes hep-ph/0412322

~ g^4: Korthals-Altes, Laine, Romatschke 08...

How color evaporates in the semi-QGP

AMMPV: simple trick.

$$\mathrm{tr}\ \frac{1}{\mathrm{e}^{(E-iQ^a)/T}-1} = \mathrm{tr}\ \sum_{j=1}^{\infty} \mathrm{e}^{-j(E-iQ^a)/T} = \sum_{j=1}^{\infty} \mathrm{e}^{-jE/T}\ \mathrm{tr}\ \mathbf{L}^j$$

L = $e^{i\,Q/T}$ = Wilson line. Obtain expressions in terms of moments of L, L^j.

We *don't* know (yet) effective theory for Q's. *So we guess.*

Take first moment, *l* = <*loop*> = < tr L>/N, from lattice for N = 3.
For higher moments, given *l*, assume either: 1. Gross-Witten, or 2. step function.

L ~ propagator of *infinitely* heavy (test) quark.
In *this* semi-cl. expansion, for colored fields of *any* momentum and mass,
As $l \to 0$, *all* quarks suppressed ~ *l* ; *all* gluons, ~ l^2 : *universal* color evaporation

Smells right: *all* colored fields *should* evaporate as <*loop*> → 0.

Shear viscosity in the semi-QGP

Shear viscosity, η, in the complete QGP:

Arnold, Moore & Yaffe, hep-ph/0010177 & 0302165 = AMY.

Generalize to Q ≠ 0: Boltzmann equation in background field.

$$\eta = \frac{\mathcal{S}^2}{\mathcal{C}}$$

S = source, C = collision term. *Two* ways of getting small η:

"Strong" QGP, *large* coupling $S \sim 1$, $C \sim (\text{coupling})^2 >> 1$.

$\mathcal{N} = 4$ SU(N), $g^2 N = N = \infty$: $\eta/s = 1/4\pi$. Kovtun, Son & Starinets hep-th/0405231

"Semi" QGP: small *loop* at *moderate* coupling:

Pure glue: $S \sim \langle loop \rangle^2$, $C \sim g^4 \langle loop \rangle^2$

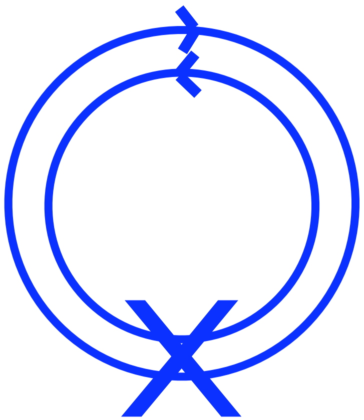
With quarks: $S \sim \langle loop \rangle$, $C \sim g^4$

Both: $\eta \sim \langle loop \rangle^2$

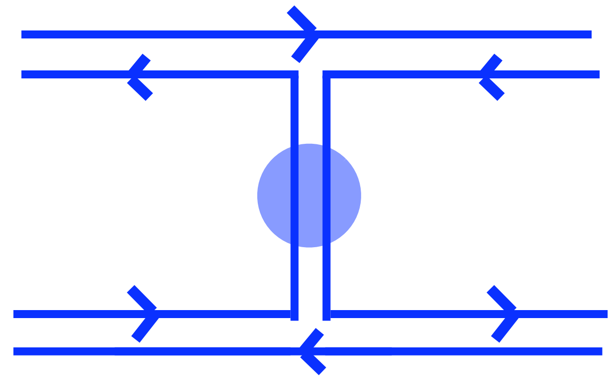
To leading log order: # from AMY, constant "c" beyond leading log

$$\frac{\eta}{T^3} = \frac{\#}{g^4 \log(c/g)}\, \mathcal{R}(\ell) \quad ; \quad \mathcal{R}(\ell \to 0) \sim \ell^2$$

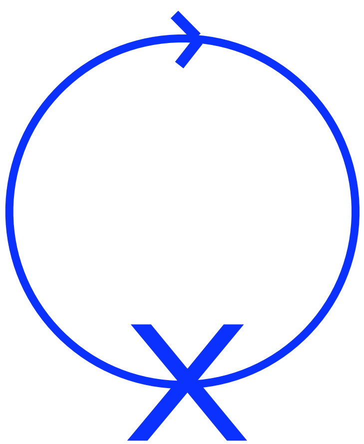
Counting powers of <*loop*> = *l* → 0



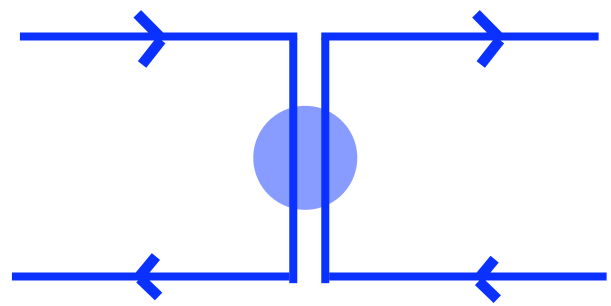
$\mathcal{S} \sim \ell^2$



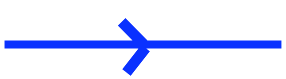
$\mathcal{C} \sim \ell^2$



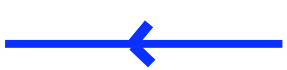
$\mathcal{S} \sim \ell$



$\mathcal{C} \sim 1$



$\sim e^{+iQ^a/T}$



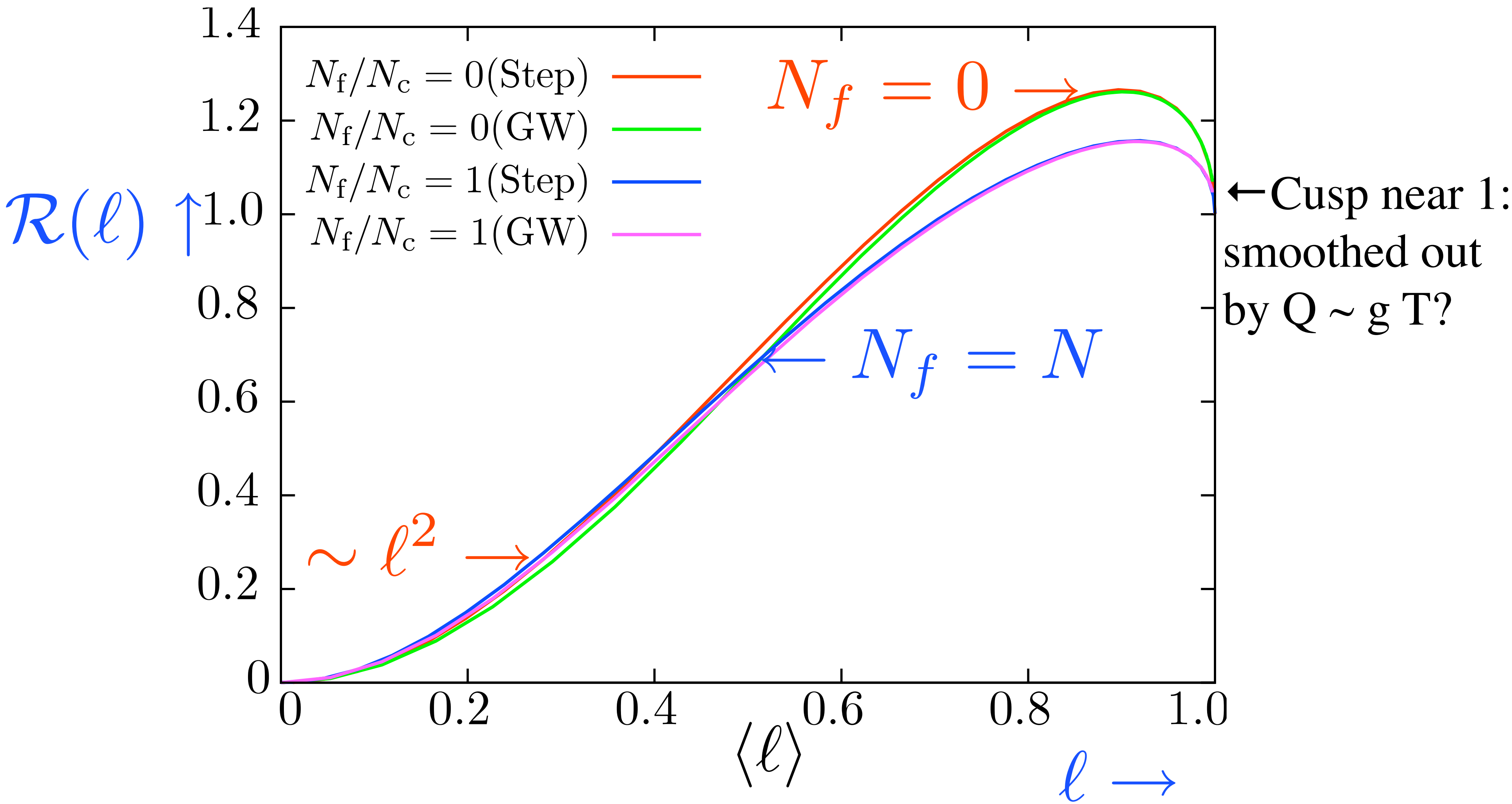
$\sim e^{-iQ^a/T}$

Small shear viscosity from color evaporation

R = ratio of shear viscosity in semi-QGP/complete-QGP at *same* g, T.

Two different eigenvalue distributions give *very* similar results!

When <*loop*> ~ 0.3, *R* ~ 0.3.

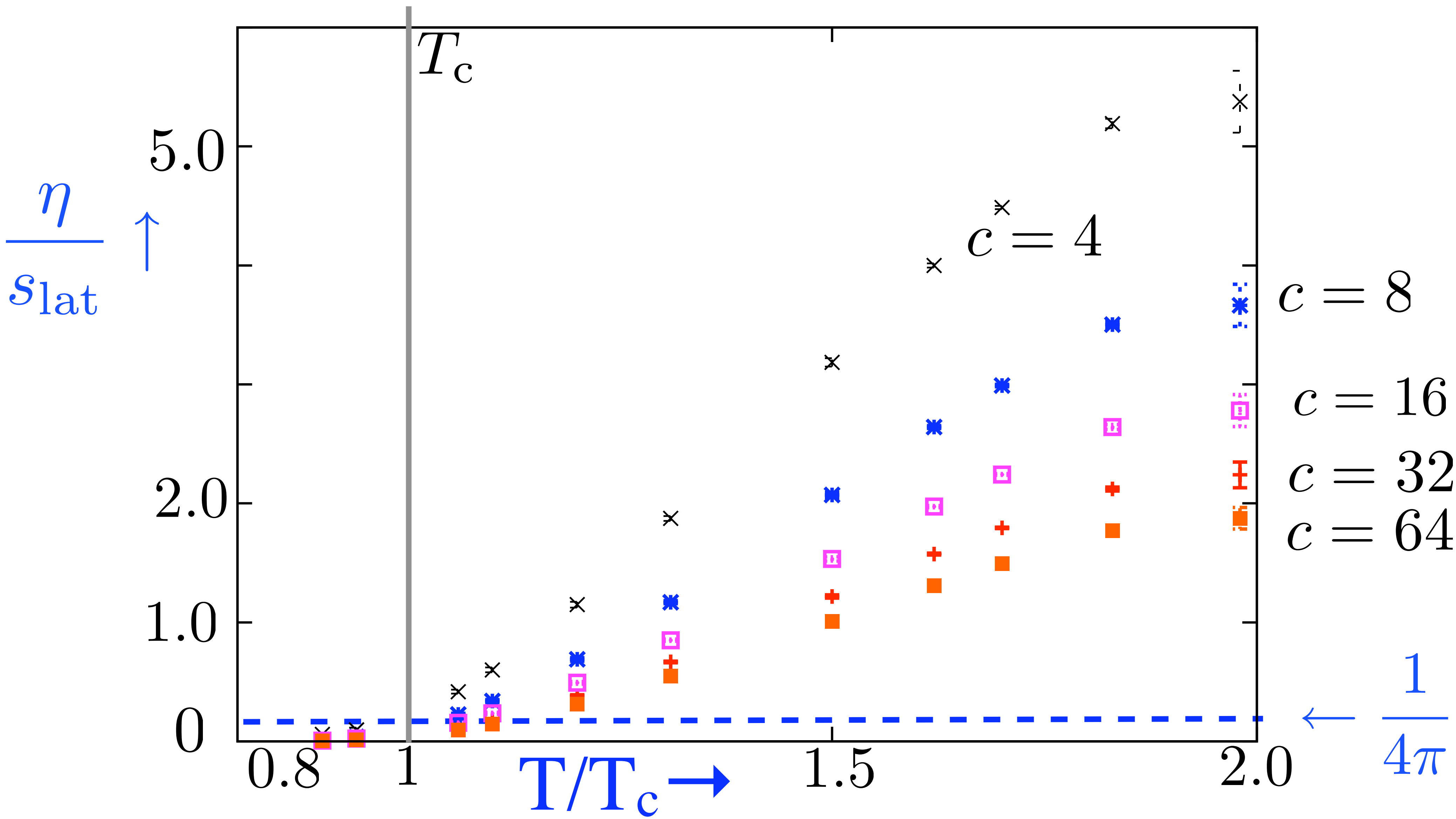


Shear viscosity/entropy

Leading log shear viscosity/lattice entropy. $\alpha_s(T_c) \sim 0.3$.

Large increase from T_c to 2 T_c. *Clearly* need results beyond leading log.

Also need to include: quarks and gluons *below* T_c, hadrons *above* T_c. *Not* easy.



Strong- vs. Semi-QGP at the LHC

At RHIC, $\eta/s \sim 0.1 \pm 0.1$
Luzum & Romatschke, 0804.4015

Close to $\mathcal{N} = 4$ SU(∞), $\eta/s = 1/(4\pi)$.

Strong-QGP: in $\mathcal{N} = 4$ SU(∞),
add scalar potential to fit lattice pressure
But η/s *remains* $= 1/4\pi$!
Evans & Threlfall, 0805.0956
Gubser & Nellore, 0804.0434
Gursoy, Kiritsis, Mazzanti & Nitti 0804.0899
So LHC nearly ideal, like RHIC.

Semi-QGP, *and* non-relativistic systems →
Large change in η/s from T_c to $2\,T_c$.
At *early* times, LHC *viscous*,
*un*like RHIC

Lacey, Ajitnand, Alexander, Chung, Holzman, Issah, Taranenko, Danielewicz & Stocker, nucl-ex/0609025 ↓